Surface Areas and Volume of Solids

Ex No: 25.1

Solution 1.

Let the side of the cube be 'a' cm

$\therefore$ Volume of cube = a^3

$\therefore$ $1331 = a^3$ (Given)

$\therefore$ $a = \sqrt[3]{1331}$ (Taking cube roots on both sides)

$\therefore$ a = 11 cm

Surface area of a cube = $6a^2$

$= 6 \times 11^2$

$= 6 \times 121$

$= 726 \text{ cm}^2$

Hence, the surface area of the cube is 726 cm^2.

Solution 2.

Let the side of the cube be 'a' cm

$\therefore$ Total surface area of a cube = $6a^2$

$\therefore$ $864 = 6a^2$ (Given)

$a^2 = \frac{864}{6}$

$a^2 = 144$

$a = \sqrt{144}$ (Taking square roots on both sides)

$\therefore$ a = 12 cm

Volume of a cube = a^3

$= 12^3$

$= 1728 \text{ cm}^3$

$\therefore$ Volume of the cube = 1728 cm^3.

Solution 3.

Let the length, breadth and height of the rectangular solid be 'a', 'b' and 'c' respectively.

$\therefore$ a : b : c = 6 : 4 : 3 (Given)

Let the common multiple be x

a = 6x cm

b = 4x cm

c = 3x cm

The total surface area of the cuboid = 1728 cm^2 (Given)

$$2(lb + bh + hl) = 1728$$

$$2(ab + bc + ca) = 1728$$

$$2[(6x.4x) + (4x.3x) + (3x.6x)] = 1728$$

$$24x^2 + 12x^2 + 18x^2 = \frac{1728}{2}$$

$$54x^2 = 864$$

$$x^2 = \frac{864}{54}$$

$$x^2 = 16$$

$$x = \sqrt{16}$$

$$\therefore x = 4$$

$\therefore$ a = 6x = 6 x 4 = 24 cm

$\therefore$ b = 4x = 4 x 4 = 16 cm

$\therefore$ c = 3x = 3 x 4 = 12 cm

Hence, the dimensions of the rectangular solid are 24 cm, 16 cm and 12 cm.

Solution 4.

Given that:

Diagonal of a cube = $\sqrt{48}$ cm

i.e., $\sqrt{3} \times l = \sqrt{48}$ [$\because$ Diagonal of cube = $\sqrt{3} \times l$]

$$l = \frac{\sqrt{48}}{\sqrt{3}}$$

$$l = \sqrt{\frac{48}{3}}$$

$$= \sqrt{16}$$
$$= 4 \text{ cm}$$

$\therefore$ Side (l) = 4 cm

Now,

Volume of cube = l^3
$= l \times l \times l$
$= 4 \times 4 \times 4$
$= 16 \times 4$
$= 64 \text{ cm}^3$

$\therefore$ Volume of Cube = 64 cm^3

Solution 5.

Volume of a cuboid = l x b x h

2400 = 20 × 15 × h

h = 8 cm

Hence, height of the cuboid is 8 cm.

Solution 6.

Given that: Diagonal of cuboid = $3\sqrt{29}$ cm .... (1)

Ratio of Length, breadth & height = 2:3:4

$\therefore$ Length (l) = 2x
Breadth (b) = 3x &
Height (h) = 4x

We know that: Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$

$$= \sqrt{(2x)^2 + (3x)^2 + (4x)^2}$$
$$= \sqrt{4x^2 + 9x^2 + 16x^2}$$
$$= \sqrt{29x^2}$$
$$= x\sqrt{29}$$

Also,

$$x\sqrt{29} = 3\sqrt{29} \quad \text{[From (1)]}$$

i.e.,

$$x = 3\frac{\sqrt{29}}{\sqrt{29}}$$

$\therefore$ x = 3 cm

Thus,

Length = 2 x 3 = 6 cm
Breadth = 3 x 3 = 9 cm
Height = 4 x 3 = 12 cm

$\therefore$ Volume of cuboid = l x b x h
= 6 x 9 x 12
= 54 x 12
= 648 cm^3

Solution 7.

Given that:

T.S.A. of cube = 294 cm^2

i.e., $6 \times l \times l = 294$ [$\because$ T.S.A. of cube = $6 \times l^2$]

$$l^2 = \frac{294}{6}$$

$$l^2 = 49$$

$$l = \sqrt{49} = 7 \text{ cm}$$

$\therefore$ Side (l) = 7 cm

$\therefore$ Volume of cube = $l \times l \times l$
$= 7 \times 7 \times 7$
$= 343 \text{ cm}^3$

Solution 8.

Given that:

T.S.A. of a cuboid = 46 m^2 .... (1)
Height = 1 m
Breadth = 3 m

Let the length of cuboid = l m

We know that:-
T.S.A. of cuboid = $2 \times \{(l \times b)+(b \times h)+(h \times l)\}$ .... (2)

On comparing (1) & (2) we get,

$$2 \times \{(l \times b)+(b \times h)+(h \times l)\} = 46$$
$$2 \times \{(l \times 3)+(3 \times 1)+(1 \times l)\} = 46$$
$$2 \times \{3l+3+l\} = 46$$
$$2 \times \{4l+3\} = 46$$
$$8l+6 = 46$$
$$8l = 46 - 6$$
$$8l = 40$$
$$l = \frac{40}{8}$$
$$l = 5 \text{ m}$$

$\therefore$ Length (l) = 5 m

Now,
Volume of cuboid = $l \times b \times h$
$= 5 \times 3 \times 1$
$= 15 \text{ m}^3$

Solution 10.

Given that:

Diagonal of cuboid = $5\sqrt{34}$ cm .... (1)

Ratio of Length, breadth & height = 3:3:4

$\therefore$ Length (l) = 3x
Breadth (b) = 3x &
Height (h) = 4x

We know that:-

$$\text{Diagonal of cuboid} = \sqrt{l^2 + b^2 + h^2}$$
$$= \sqrt{(3x)^2 + (3x)^2 + (4x)^2}$$
$$= \sqrt{9x^2 + 9x^2 + 16x^2}$$
$$= \sqrt{34x^2}$$
$$= x\sqrt{34}$$

Also,

$$x\sqrt{34} = 5\sqrt{34} \quad \text{[From (1)]}$$

i.e.,
$$x = 5\frac{\sqrt{34}}{\sqrt{34}}$$

$$\therefore \quad x = 5 \text{ cm}$$

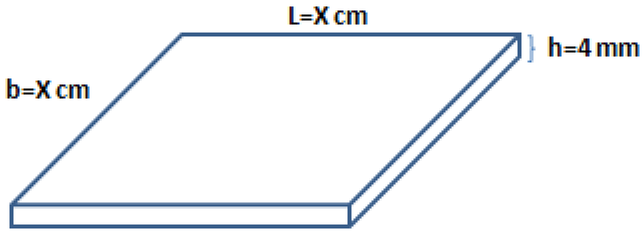
Thus,

Length = 3 x 5 = 15 cm
Breadth = 3 x 5 = 15 cm
Height = 4 x 5 = 20 cm

$$\therefore \text{Volume of cuboid} = l \times b \times h$$
$$= 15 \times 15 \times 20$$
$$= 225 \times 20$$
$$= 4500 \text{ cm}^3$$
$$= 0.0045 \text{ m}^3 \quad [\because 1 \text{ m}^3 = 10^6 \text{cm}^3]$$

Solution 11.

Volume of the square plate = Volume of a cuboid



$$h = 4 \text{ mm} = \frac{4}{10} \text{ cm} = 0.4 \text{ cm}$$

Volume of the square plate = l x b x h

$$1440 = x \times x \times 0.4$$

$$1440 = x^2 \times 0.4$$

$$x^2 = \frac{1440}{0.4} = 3600$$

$$x = \sqrt{3600}$$

$$\therefore x = 60\text{cm}$$

Solution 12.

Given that:

Length (l) of room = 22 m
Breadth (b) of room = 15 m &
Height (h) of room = 6 m

L.S.A of room = 2 x h x (l + b)
= 2 x 6 x (22 + 15)
= 2 x 6 x 37
= 444 m^2

$\therefore$ Area of its 4 walls = 444 m^2

Cost of painting the walls = 12 per m^2
i.e., for 1 m^2 = Rs 12

$\therefore$ For 444 m^2 = Rs 12 x 444
= Rs 5328

Solution 13.

Given that:

Length of the cuboid = 25 cm
Breadth of the cuboid = 15 cm
Height of the cuboid = 9 cm &
Volume of cube = volume of cuboid

We know that volume of Cuboid = l x b x h
= 25 x 15 x 9
= 3375 cm^3

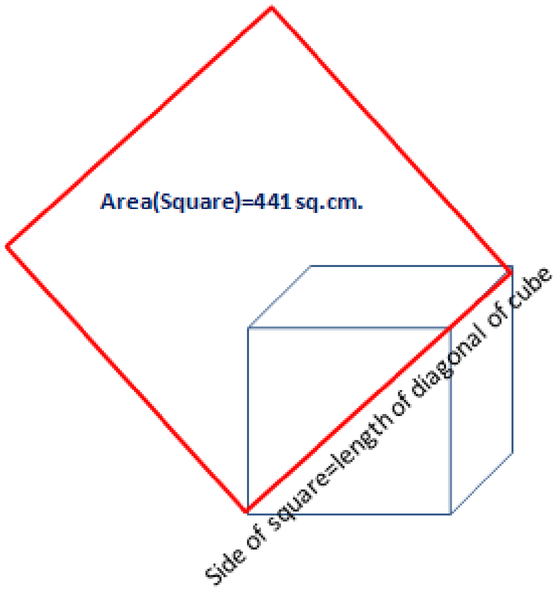
$\therefore$ Volume of cube = 3375 cm^3
But we know that volume of cube = l^3

i.e., $l^3 = 3375$
$l = \sqrt[3]{3375}$
= 15 cm

$\therefore$ Side (l) = 15 cm

Now,
T.S.A of cube = $6l^2$
= 6 x 15 x 15
= 1350 cm^2

Solution 14.



Area of a square = 441 cm^2

$$side^2 = 441$$

$$side = \sqrt{441}$$

$$\therefore\ side = 21cm$$

$\therefore$ The length of the diagonal of the cube is 21 cm.

Let 'a' be the side of the cube

Diagonal of a cube = $\sqrt{3} \times a$

$$\therefore 21 = \sqrt{3} \times a$$

$$a = \frac{21}{\sqrt{3}}$$

$$a = \frac{21}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

(rationali sin g the denominator)

$$a = \frac{21\sqrt{3}}{3}$$

$$\therefore\ a = 7\sqrt{3}\ cm$$

Total surface area of a cube = $6a^2$

$$= 6 \times (7\sqrt{3})^2$$

$$= 6 \times 49 \times 3$$

$$= 882\ cm^2$$

$\therefore$ Side of the cube is $7\sqrt{3}$ cm and the total surface area of the cube is 882 cm^2.

Solution 15.

Given that

Side (l_1) of metal cube (a) = 6 cm

Side (l_2) of metal cube (b) = 8 cm

Side (l_3) of metal cube (c) = 10 cm

Total Volume of all three cubes = Volume of 1 cube

Volume of cube (a) = $(l_1)^3 = 6^3 = 216$ cm^3

Volume of cube (b) = $(l_2)^3 = 8^3 = 512$ cm^3

Volume of cube (c) = $(l_3)^3 = 10^3 = 1000$ cm^3

Total volume of all three cubes = 1728 cm^3

$\therefore$ Volume of 1 cube= 1728 cm3

i.e., $l^3 = 1728$

$l = \sqrt[3]{1728}$

$\therefore$ Side(l) = 12 cm

Length of diagonal of cube = $\sqrt{3} \times l$

$= \sqrt{3} \times 12$

$= 12\sqrt{3}$ cm

Solution 16.

Given that:-

Side (l_1) of cube (a) = x cm

Side (l_2) of cube (b) = 8 cm

Side (l_3) of cube (c) = 10 cm

Edge length of new formed cube = 12 cm

Volume of cube (a) = $(l_1)^3 = x^3$ cm^3

Volume of cube (b) = $(l_2)^3 = 8^3 = 512$ cm^3

Volume of cube (c) = $(l_3)^3 = 10^3 = 1000$ cm^3

Total Volume of all three cubes = Volume of 1 cube

$$x^3 + 8^3 + 10^3 = 12^3$$

$$x^3 + 512 + 1000 = 1728$$

$$x^3 = 1728 - 1512$$

$$x = \sqrt[3]{216}$$

$$\therefore \quad x = 6 \text{ cm}$$

Solution 17.

Given that:

Three cubes of equal side (l) = 5 cm

Volume of 1 cube = l^3

$= 5^3$

$= 125\ cm^3$

Volume of 3 cubes = 125 x 3

$= 375\ cm^3$

Since, Volume of 3 cubes = Volume of cuboid

∴ Volume of cuboid = $375\ cm^3$

Now,

When the cubes are joined together, the breadth and height of the new cuboid Formed remains the same whereas length changes.

Length of each cube=5cm

∴ Length (l) of 3 cubes joined together=3 x 5cm=15cm

Breadth (b) of the new cuboid=5cm

Height (h) of the new cuboid =5cm

∴ T.S.A of the cuboid= 2 x {(l x b)+(b x h)+(h x l)}

=2{(15 x 5)+(5 x 5) +(5 x 15)}

=2{75+25+75}

=2 x 175

∴ T.S.A of cuboid = $350\ cm^2$

Solution 21.

We need to find: $\dfrac{\text{Total surface area of cuboid}}{\text{Sum of total surface areas of 3 cubes}}$

Cube:

Let the side of the cube be 'a' units

$\therefore$ Total surface area of 1 cube = $6a^2$ sq. units

$\therefore$ Total surface area of 3 such cubes = $3 \times 6a^2$ sq. units

$= 18a^2$ sq. units

The cuboid is formed by joining 3 cubes:

length = 3a cm

breadth = a cm

height = a cm

$\therefore$ Total surface area of cuboid = $2(lb + bh + hl)$

$= 2(3a \times a + a \times a + a \times 3a)$

$= 2(3a^2 + a^2 + 3a^2)$

$= 2(7a^2)$

$= 14a^2$ sq. units

$$\frac{\text{Total surface area of cuboid}}{\text{Sum of total surface areas of 3 cubes}} = \frac{14a^2}{18a^2} = \frac{7}{9}$$

$\therefore$ The ratio of Total surface area of cuboid to the Sum of total surface areas of 3 cubes is 7 : 9.

Solution 22.

Volume of metal cube = Volume of water level risen in the tank

l^3 = Volume of water level risen in the tank

$\therefore$ Volume of water level risen in the tank $= 4^3 = 64\ cm^3$

The volume of water rise is in the shape of the cuboid

$\therefore l \times b \times h = 64$

$8 \times 4 \times h = 64$

$\therefore h = \dfrac{64}{8 \times 4}$

$\therefore h = 2$ cm

$\therefore$ The rise in the water level is 2 cm.

Solution 23.

Volume of the metal = l x b x h

$= 6 \times 5 \times x$

$= 30x \text{ cm}^3$

Volume of water risen = Volume of metal immersed

$l \times b \times h = 30x$

$18 \times 8 \times 0.5 = 30x$

$72 = 30x$

$x = 2.4 \text{ cm}$

Solution 24.

Thickness of the closed box = 5 mm = 0.5 cm

External Dimensions are:

length = 21 cm

breadth = 13 cm

height = 11 cm

Internal dimensions = External dimensions - 2(thickness)

∴ Internal Dimensions are:

length = 20 cm

breadth = 12 cm

height = 10 cm

Volume of the wood used in making the box

= Volume of External cuboid - Volume of internal cuboid

$= (21 \times 13 \times 11) - (20 \times 12 \times 10)$

$= 3003 - 2400$

$= 603 \text{ cm}^3$

Hence, the volume of wood used in making the box is 603 cm^3.

Solution 26.

Given that

Length (l) of the room = 5 m

Breadth (b) of the room = 2 m

Height (h) of the room = 4 m

$\therefore$ Volume of air in the room = l x b x h

= 5 x 2 x 4

= 40 m^3

ince, 1 person needs = 0.16 m^3 of air

e., 0.16 m^3 of air = 1 person

$\therefore$ 1 m^3 of air = $\frac{1}{0.16}$ person

So, 40 m^3 of air = $\frac{1}{0.16}$ x 40

$= \frac{40}{0.16} \times \frac{100}{100}$

$= \frac{4000}{16}$

= 250 Persons

Thus, the room can accommodate 250 persons.

Solution 27.

Given that:

No. of persons accommodated in a room = 375

Ratio of dimensions of room = 6:4:1

∴ Length (l) of the room = 6x m

Breadth (b) of the room = 4x m

Height (h) of the room = x m

Air consumed by 1 person = 64 m^3

∴ Air consumed by 375 persons = 64 x 375

= 24,000 m^3

i.e., Volume of air in the room = 24,000 m^3 .... (1)

Also,

Volume (V) of room is given by:-

$$V = l \times b \times h$$

Substituting in (1) we get,

$$l \times b \times h = 24{,}000$$
$$6x \times 4x \times x = 24{,}000$$
$$24x^3 = 24{,}000$$
$$x^3 = \frac{24000}{24}$$
$$x = \sqrt[3]{1000}$$
$$x = 10 \text{ m}$$

∴ Length (l) of the room = 6 x 10 = 60 m

Breadth (b) of the room = 4 x 10 = 40 m

Height (h) of the room = 1 x 10 = 10 m

Now,

L.S.A of the room = 2 x h x (l+ b)

= 2 x 10 x (60 + 40)

= 20 x 100

= 2000 m^2

i.e., Area of the 4 walls of the room = 2000 m^2

Solution 28.

Given that:

Dimensions of the class room:

Length (l_1) of the room = 7 m

Breadth (b_1) of the room = 6 m

Height (h_1) of the room = 4 m

Dimensions of the doors:

Length (l_2) = 3 m

Breadth (b_2) = 1.4 m

No. of doors = 2

Dimensions of the windows:

Length (l_3) = 2 m

Breadth (b_3) = 1 m

No. of windows = 6

Area of doors $= (l_2 \times b_2) \times 2$
$= (3 \times 1.4) \times 2$
$= 4.2 \times 2$
$= 8.4\ m^2$

Area of windows $= (l_3 \times b_3) \times 6$
$= (2 \times 1) \times 6$
$= 2 \times 6$
$= 12\ m^2$

Now,

T.S.A of the room $= 2 \times \{(l_1 \times b_1) + (b_1 \times h_1) + (h_1 \times l_1)\}$
$= 2 \times \{(7 \times 6) + (6 \times 4) + (4 \times 7)\}$
$= 2 \times \{42 + 24 + 28\}$
$= 2 \times 94$
$= 188\ m^2$

Since the inner walls of the room has to be painted,

. Total area to be painted = T.S.A of the room – (Ar. of doors) – (Ar. of windows)
$= 188 - 8.4 - 12$
$= 179.6 - 12$
$= 167.6\ m^2$

Cost of colouring $1 m^2$ area = Rs 15

$\therefore$ Cost of colouring $167.6\ m^2$ area of the wall = Rs 15×167.6
= Rs 2514

Solution 30

$\because$ The cost of papering the four walls of the room at Rs 1 per m^2 is Rs. 210.

$\therefore$ The area of the 4 walls is 210 m^2.

The length and breadth are in the ratio 5 : 2 (given)

Let the common multiple be x

$\therefore$ Length = 5x m

breadth = 2x m

height = 5 m (given)

Surface area of the 4 walls = 2hl+2hb

$$210 = 2 \times 5 \times 5x + 2 \times 5 \times 2x$$

$$210 = 50x + 20x$$

$$210 = 70x$$

$$\therefore x = \frac{210}{70}$$

$$\therefore x = 3$$

length = 5x = 5 x 3 = 15 cm

breadth = 2x = 2 x 3 = 6 cm

Solution 32.

Thickness of the closed box = 1 cm

External Dimensions are:

l = 22 cm
b = 18 cm
h = 14 cm
Internal dimensions = External dimensions -2(thickness)

Internal dimensions are:

l = 20 cm
b = 16 cm
h = 12 cm

Volume of wood used in making the box
= Volume of External cuboid - Volume of internal cuboid
$= (22 \times 18 \times 14) - (20 \times 16 \times 12)$
= 5544 - 3840
$= 1704\ cm^3$

$\therefore$ The volume of the wood used in making the box is $1704\ cm^3$. (Ans 1)

The cost of the wood required to make the box at the rate of Rs. 5 per cm^3

$= 5 \times 1704$

$=$ Rs. 8520 (Ans 2)

Side of the cube = 2 cm

$\therefore$ Volume of the cube = 8 cm^3

Volume of box from inside = Volume of internal cuboid

$= 20 \times 16 \times 12$

$= 3840\ cm^3$

$\therefore$ The no. of cubes that can fit inside the box

$$= \frac{\text{Volume of internal cuboid}}{\text{Volume of each small cube}}$$

$$= \frac{3840}{8}$$

$= 480$ cubes (Ans 3)

Solution 33.

Given that:

The length of a cold storage is double its breadth

Height = 3 m

Area of its four walls = $108\ m^2$ .... (1)

Let the Breadth (b) of cold storage = x m

Thus, the length (l) of cold storage = 2x m

$$\text{L.S.A of cold storage} = 2 \times h \times (l + b)$$

$$108 = 2 \times 3 \times (2x + x) \qquad \text{[From (1)]}$$

$$= 6 \times 3x$$

$$18x = 108$$

$$x = \frac{108}{18}$$

$$x = 6$$

$\therefore$ Length (l) = 2 x 6 = 12 m

Breadth (b) = 1 x 6 = 6 m

Thus,

$$\text{Volume of cold storage} = l \times b \times h$$

$$= 12 \times 6 \times 3$$

$$= 216\ m^3$$

Solution 34.

Given that
Dimensions of metallic sheet:

Length (l) = 48 cm
Breadth (b) = 36 cm

Side (S) of each square = 8 cm.

Now,

Area of metallic sheet = l x b
= 48 x 36
$= 1728 \text{ cm}^2$

Area of 1 square = S x S
= 8 x 8
$= 64 \text{ cm}^2$

$\therefore$ Area of 4 squares = 64 x 4
$= 256 \text{ cm}^2$

Thus, remaining area in the sheet after reducing the area of 4 squares:

Remaining area = 1728 – 256
$= 1472 \text{ cm}^2$ .... (1)

Since 8 cm square is cut off from all sides, we get the dimensions of open box as:

Length (l) = 48 – 16 = 32 cm
Breadth (b) = 36 – 16 = 20 cm

Area of the box = L.S.A of the box + area of base of the box
1472 = {2 x h x (l+ b)} + (l x b) [From (1)]
1472 = {2 x h x (32 + 20)} + (32 x 20)
1472 = {2h x 52} + 640
1472 = 104h + 640
104h = 1472 – 640

$$h = \frac{832}{104}$$

i.e., height (h) = 8 cm

Thus, volume of the box = l x b x h
= 32 x 20 x 8
$= 5120 \text{ cm}^3$

Solution 35.

Given that:

Area of playground = 4800 m^2

i.e., $l \times b = 4800\ m^2$ .... (1)

Height (h) = 2.5 cm
= 0.025 m [$\because$ 1 m = 100 cm]

Volume of playground = $l \times b \times h$
= 4800×0.025 [From (1)]
= 120 m^3

Cost of gravel = Rs 7.25 per m^3

i.e., For 1 m^3 = Rs 7.25

$\therefore$ For 120 m^3 = Rs 7.25×120
= Rs 870

Thus the cost of covering the area with gravel = Rs 870.

Solution 36.

Volume of the fall in the level of water in the rectangular tank = Volume of cube

$$\therefore l \times b \times h = \text{side}^3$$

$$9 \times 6 \times h = 3^3$$

$$h = \frac{27}{9 \times 6}$$

$$\therefore h = 0.5 \text{ cm}$$

$\therefore$ The fall in the level of water in the container is 0.5 cm.

Solution 39.

When the cube is submerged, the level of water is increased and some water flows out of it.

Volume of the cube
= Volume of water level rise + Volume of water overflown
$= 12 \times 12 \times 2 + 224$
$= 512\ cm^3$
$\therefore$ The volume of the cube is $512\ cm^3$. (Ans 1)

Volume of the cube $= s^3$

$$512 = s^3$$
$$s = \sqrt[3]{512}$$
$$s = 8\ cm$$

Surface area of cube $= 6s^2$

$$= 6 \times 8^2$$
$$= 384\ cm^2$$

$\therefore$ The surface area of the cube is $384\ cm^2$. (Ans 2)

Ex No: 25.2

Solution 1.

(i) Given that:
Radius (r) = 4.2 cm
Height (h) = 12 cm

We know that:
Lateral surface Area (L.S.A) of cylinder = $2 \times \pi \times r \times h$

Total surface Area (T.S.A) of cylinder = $(2 \times \pi \times r \times h) + (2 \times \pi \times r^2)$

Volume of cylinder = $\pi \times r^2 \times h$

L.S.A of cylinder = $2 \times \pi \times r \times h$

$= 2 \times \frac{22}{7} \times 4.2 \times 12$

$= 316.8\ cm^2$

T.S.A of cylinder = $(2 \times \pi \times r \times h) + (2 \times \pi \times r^2)$

$= (2 \times \frac{22}{7} \times 4.2 \times 12) + (2 \times \frac{22}{7} \times 4.2^2)$

$= 316.8 + 110.88$

$= 427.68\ cm^2$

Volume of cylinder = $\pi \times r^2 \times h$

$= \frac{22}{7} \times 4.2^2 \times 12$

$= 665.28\ cm^3$

(ii) Given that:
Diameter=10m
Radius (r) = 5 m
Height (h) = 7 m

Now,

L.S.A of cylinder $= 2 \times \pi \times r \times h$

$= 2 \times \frac{22}{7} \times 5 \times 7$

$= 220\ m^2$

T.S.A of cylinder $= (2 \times \pi \times r \times h) + (2 \times \pi \times r^2)$

$= (2 \times \frac{22}{7} \times 5 \times 7) + (2 \times \frac{22}{7} \times 5^2)$

$= 220 + 157.14$

$= 377.14\ m^2$

Volume of cylinder $= \pi \times r^2 \times h$

$= \frac{22}{7} \times 5^2 \times 7$

$= 550\ m^3$

Solution 2.

Diameter of cylinder = 20 cm

$\therefore$ Radius (r) $= \frac{20}{2} = 10$ cm

Let h be the height of the cylinder

Area of curved surface = 1100 cm^2

i.e, L.S.A of cylinder = 1100 cm^2

$2 \times \pi \times r \times h = 1100$ [$\because$ L.S.A of cylinder = $2 \times \pi \times r \times h$]

$$2 \times \frac{22}{7} \times 10 \times h = 1100$$

$$\frac{440}{7} h = 1100$$

$$h = \frac{1100 \times 7}{440}$$

$$h = \frac{70}{4} = 17.5 \text{ cm}$$

Thus, volume of cylinder = $\pi \times r^2 \times h$

$$= \frac{22}{7} \times 10^2 \times 17.5$$

$$= 5500 \text{ cm}^3$$

Solution 3.

Diameter of base = 14 cm

$\therefore$ Radius (r) = 7 cm

Height (h) = 24 cm

Volume of cylinder = $\pi \times r^2 \times h$

$$= \frac{22}{7} \times 7^2 \times 24$$

$$= 3696 \text{ cm}^3$$

Solution 4.

Let r be the radius of the cylinder.

Then, Height (h) = 5 + r .... (1)

Total surface area of cylinder = 264 m^2

$(2 \times \pi \times r \times h) + (2 \times \pi \times r^2) = 264$

$2 \pi r (h + r) = 264$

$2 \pi r (5 + r + r) = 264$ [From (1)]

$r (5 + 2r) = \frac{264}{2\pi}$

$5r + 2r^2 = 42$

$2r^2 + 5r - 42 = 0$

$2r^2 + 12r - 7r - 42 = 0$

$2r(r + 6) - 7(r + 6) = 0$

$(2r - 7)(r + 6) = 0$

i.e., $(2r - 7) = 0$ or $(r + 6) = 0$

$2r = 7$ or $r = -6$

$r = \frac{7}{2}$

$r = 3.5$ m

Since, radius of a cylinder cannot be negative, we take the value of r = 3.5 m

$\therefore$ Height (h) = 5 + 3.5

= 8.5 m

Volume = $\pi \times r^2 \times h$

$= \frac{22}{7} \times 3.5^2 \times 8.5$

$= 327.25$ m^3

Solution 5.

L.S.A. of cylinder = 198 cm^2

Diameter of base = 21 cm

$\therefore$ Radius (r) = 10.5 cm

Let h be the height of the cylinder

$$\text{L.S.A.} = 198\ cm^2$$

$$2 \times \pi \times r \times h = 198 \qquad [\because \text{L.S.A of cylinder} = 2 \times \pi \times r \times h]$$

$$2 \times \frac{22}{7} \times 10.5 \times h = 198$$

$$\frac{462}{7}h = 198$$

$$h = \frac{198 \times 7}{462}$$

$$h = \frac{1386}{462} = 3\ cm$$

Volume of cylinder $= \pi \times r^2 \times h$

$$= \frac{22}{7} \times 10.5^2 \times 3$$

$$= 1039.5\ cm^3$$

Solution 6.

Volume of cylinder = 7700 cm^3

Diameter of base = 35 cm

$\therefore$ Radius (r) = 17.5 cm

Let h be the height of the cylinder

$$\text{Volume} = 7700$$
$$\pi \times r^2 \times h = 7700$$
$$\frac{22}{7} \times 17.5^2 \times h = 7700$$
$$962.5h = 7700$$
$$h = \frac{7700}{962.5} \times \frac{10}{10}$$
$$h = \frac{77000}{9625}$$
$$h = 8 \text{ cm}$$

Now,

$$\text{T.S.A. of cylinder} = (2 \times \pi \times r \times h) + (2 \times \pi \times r^2)$$
$$= (2 \times \frac{22}{7} \times 17.5 \times 8) + (2 \times \frac{22}{7} \times 17.5^2)$$
$$= 880 + 1925$$
$$= 2805 \text{ cm}^2$$

Solution 7.

T.S.A. of cylinder = $3872\ cm^2$

Let r and h be the radius and height of the cylinder respectively.

Circumference of the base = 88 cm

i.e., $2 \times \pi \times r = 88$

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{44}$$

$$= 14\ cm$$

T.S.A. of cylinder = $3872\ cm^2$

$$(2 \times \pi \times r \times h) + (2 \times \pi \times r^2) = 3872$$

$$(2 \times \frac{22}{7} \times 14 \times h) + (2 \times \frac{22}{7} \times 14^2) = 3872$$

$$88h + 1232 = 3872$$

$$88h = 2640$$

$$h = \frac{2640}{88}$$

$$h = 30\ cm$$

Thus, Volume of cylinder = $\pi \times r^2 \times h$

$$= \frac{22}{7} \times 14^2 \times 30$$

$$= 18480\ cm^3$$

Solution 9.

Let the radius of base of the original cylinder = r

And the height of the cylinder = h

Volume of original cylinder = $\pi r^2 h$

Given that, the radius of new cylinder = 2r

And, height = $\frac{h}{2}$

$\therefore$ Volume of new cylinder = $\pi \times (2r)^2 \times \frac{h}{2}$

$$= 2\pi r^2 h$$

Ratio of volume of new cylinder to that of original cylinder = $\frac{2\pi r^2 h}{\pi r^2 h}$

$$= 2 : 1$$

Solution 10.

Let the radius of base of the original cylinder = r
And the height of the cylinder = h

Volume of original cylinder = $\pi r^2 h$

Given that, the radius of new cylinder = 3r
And, height = 2r

$\therefore$ Volume of new cylinder = $\pi \times (3r)^2 \times 2h$
$= 18\pi r^2 h$

Ratio of volume of new cylinder to that of original cylinder = $\frac{18\pi r^2 h}{\pi r^2 h}$

$= 18 : 1$

Solution 11.

Height (h) = 8 cm
Let r be the radius of the cylinder.
Volume of cylinder = 392π cm^3
i.e., $\pi r^2 h = 392\pi$
$r^2 \times 8 = 392$
$r^2 = 49$
$r = \sqrt{49}$
$r = 7$ cm

Now,
L.S.A of cylinder = $2 \times \pi \times r \times h$
$= 2 \times \frac{22}{7} \times 7 \times 8$
$= 352$ cm^2

T.S.A of cylinder = $(2 \times \pi \times r \times h) + (2 \times \pi \times r^2)$
$= (2 \times \frac{22}{7} \times 7 \times 8) + (2 \times \frac{22}{7} \times 7^2)$
$= 352 + 308$
$= 660$ cm^2

Solution 12.

Diameter of the wire = 0.8 cm

Radius of the wire = 0.4 cm

If 4.2 g of copper = 1 cm^3 of copper

Then 22 kg copper = $\frac{22000}{4.2} cm^3$

Volume of the copper wire = Area of base x length of wire

$$\frac{22000}{4.2} = \pi r^2 \times h$$

$$\frac{22000}{4.2} = \frac{22}{7} \times 0.4^2 \times h$$

$$h = \frac{22000 \times 7}{4.2 \times 22 \times 0.4 \times 0.4}$$

$$h = 10416.7 \text{ cm}$$

$$\therefore h = 104.17 \text{ m}$$

$\therefore$ The length of the copper wire is 104.17 m.

Solution 13.

For the solid cylinder:

diameter = 4 cm

radius = 2 cm

Let its length be l.

Volume of solid cylinder = $\pi r^2 l$

$= \pi 2^2 l$

$= 4\pi l\ cm^3$

For the hollow cylinder:

Outer diameter = 10 cm

Outer radius(R) = 5 cm

Inner radius(r) = R-thickness

r = 5-0.25

r = 4.75 cm

Volume of the hollow cylinder = $\pi R^2 h - \pi r^2 h$

$= \pi h\ (5^2 - 4.75^2)$

$= \pi \times 21(25 - 22.5625)$

$= 51.1875\pi\ cm^3$

Since the solid cylinder is recast into a hollow cylinder,
Volume of solid cylinder
= Volume of material in the hollow cylinder

$4\pi l = 51.1875\pi$

$l = \frac{51.1875\pi}{4\pi}$

$l = 12.80$ cm

∴ The length of the solid cylinder is 12.80 cm.

Solution 15.

Outer diameter of roller = 30 cm

$$\text{Outer radius} = \frac{30}{2} = 15\text{ cm}$$

Thickness of iron = 2 cm

Length of roller = 1 m = 100 cm

Inner radius = Outer radius - Thickness = 13 cm

Volume of iron

$= \pi\{(r_{outer})^2 - (r_{inner})^2\} \times h$

$= \{(15)^2 - (13)^2\} \times 100$

$= 17600\text{ cm}^3$

∴ The volume of the iron is 17600 cm^3. (Ans 1)

The roller travels 8 rounds in 1 second,

∴ Total rounds made in 6 seconds = 6 x 8 = 48

In one round, distance travelled by roller

= Circumference of the curved surface

∴ Distance travelled in 6 seconds

= Circumference x 48

$= 2\pi r_{outer} \times 48$

= 4.52 m

∴ Distance travelled in 6 seconds is 4.52 m. (Ans 2)

Area covered in 6 seconds = Distance travelled x Width

Area covered = 4.52×1

$= 4.52\text{ m}^2$

∴ The area covered by the roller in 6 seconds is 4.52 m^2. (Ans 3)

Solution 16.

Dimensions of rectangle = 36 cm x 20 cm

Let the rectangle be rolled along its length to form a cylinder, thus the length and breadth of the rectangle will be equal to circumference and height (h) of the cylinder respectively.

Let r be the radius of the cylinder.

Circumference of cylinder = 36 cm

$$2 \times \pi \times r = 36$$

$$r = \frac{36}{2\pi}$$

$$r = \frac{18}{\pi} \text{ cm}$$

thus,

Volume of the cylinder so formed $= \pi \times r^2 \times h$

$$= \pi \times \left(\frac{18}{\pi}\right)^2 \times 20$$

$$= \frac{6480}{\pi} \text{ cm}^3 \quad \text{..........................} \ (1)$$

Now,

Let the rectangle be rolled along its breadth to form a cylinder, thus the length and breadth of the rectangle will be equal to height (H) and circumference of the cylinder respectively.

Let R be the radius of the cylinder.

Circumference of cylinder = 20 cm

$$2 \times \pi \times R = 20$$

$$R = \frac{20}{2\pi}$$

$$R = \frac{10}{\pi} \text{ cm}$$

thus,

Volume of the cylinder so formed $= \pi \times R^2 \times H$

$$= \pi \times \left(\frac{10}{\pi}\right)^2 \times 36$$

$$= \frac{3600}{\pi} \text{ cm}^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$\therefore$ Ratio of volumes of two cylinders $= \frac{(1)}{(2)}$

$$= \frac{6480}{3600}$$

$$= 9 : 5$$

Solution 17.

Dimensions of iron sheet = 2.2 m x 1.5 m

Let the iron sheet be rolled along its length to form a cylinder, thus the length and breadth of the sheet will be equal to circumference and height (h) of the cylinder respectively.

Let r be the radius of the cylinder

Circumference of cylinder = 2.2 m

$$2 \times \pi \times r = 2.2$$

$$r = \frac{2.2}{2\pi}$$

$$r = \frac{1.1}{\pi} \text{ m}$$

Thus,

Volume of the cylinder so formed $= \pi \times r^2 \times h$

$$= \pi \times \left(\frac{1.1}{\pi}\right)^2 \times 1.5$$

$$= \frac{1.815}{\pi} \text{ m}^3 \quad \text{.............................} \ (1)$$

Now,

Let the iron sheet be rolled along its breadth to form a cylinder, thus the length and breadth of the sheet will be equal to height (H) and circumference of the cylinder respectively.

Let R be the radius of the cylinder.

Circumference of cylinder = 1.5 m

$$2 \times \pi \times R = 1.5$$

$$R = \frac{1.5}{2\pi}$$

$$R = \frac{0.75}{\pi} \text{ m}$$

thus,

$$\text{Volume of the cylinder so formed} = \pi \times R^2 \times H$$

$$= \pi \times \left(\frac{0.75}{\pi}\right)^2 \times 2.2$$

$$= \frac{1.2375}{\pi} \text{ m}^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\therefore \text{ Ratio of volumes of two cylinders} = \frac{(1)}{(2)}$$

$$= \frac{1.815}{1.2375} \times \frac{10,000}{10,000}$$

$$= \frac{18150}{12375}$$

$$= 22 : 15$$

Solution 18.

Dimensions of rectangular strip = 36 cm x 22 cm

The rectangular strip is rotated about its length to form a cylinder, thus the length and breadth of the sheet will be equal to circumference and height (h) of the cylinder respectively.

Let r be the radius of the cylinder.

Circumference of cylinder = 36 cm

$$2 \times \pi \times r = 36 \text{ cm} \quad \ldots\ldots\ldots\ldots (1)$$

$$r = \frac{36}{2\pi}$$

$$r = \frac{18}{\pi} \text{ cm}$$

thus,

$$\text{Volume of the cylinder so formed} = \pi \times r^2 \times h$$

$$= \pi \times \left(\frac{18}{\pi}\right)^2 \times 22$$

$$= 18^2 \times 7$$

$$= 2268 \text{ cm}^3$$

Now,

$$\text{T.S.A of cylinder} = (2 \times \pi \times r \times h) + (2 \times \pi \times r \times r)$$

$$= (36 \times 22) + (36 \times \frac{18}{\pi}) \quad \text{[from (1)]}$$

$$= 792 + 206.18$$

$$= 998.18 \text{ cm}^2$$

Solution 20.

Depth or height (h) of cylindrical well = 42 m

Diameter = 14 m

$\therefore$ Radius (r) = 7 m

Volume of the well $= \pi \times r^2 \times h$

$= \frac{22}{7} \times 7^2 \times 42$

$= 6468\ m^3$

Curved surface area of walls $= 2 \times \pi \times r \times h$

$= 2 \times \frac{22}{7} \times 7 \times 42$

$= 1848\ m^2$

Cost of plastering 1 m^2 area of the wall = Rs 15

$\therefore$ Cost of plastering 1848 m^2 area of the wall = Rs 15 x 1848

= Rs 27,720

Solution 21.

The shape of the well will be cylindrical.

Depth (h_1) of well = 21 m

Diameter of well = 14 m

$\therefore$ Radius (r_1) of well = 7 m

Width of embankment = 14 m

Radius of well with embankment (r_2)=7m+14m=21m

Let height of the embankment be h_2

$\therefore$ Volume of earth spread on the embankment$= \pi h_2(r_2^2 - r_1^2) = \pi h_2(r_2 + r_1)(r_2 - r_1)$

$= \frac{22}{7} \times h_2 \times 28 \times 14 = 1232h_2$

Volume of soil dug from well = Volume of earth used to form embankment

i.e., $\pi \times r_1^2 \times h_1 = \pi h_2(r_2^2 - r_1^2)$

$\frac{22}{7} \times 7^2 \times 21 = 1232h_2$

$h_2 = \frac{3234}{1232} = 2.625$

$= 2.625\ m$

$\therefore$ Height of the embankment = 2.625 m

Solution 23.

Radius of the well (r) = $\frac{6}{2}$ = 3 m

Volume of earth dug out of the well

$= \pi r^2 h$ (Volume of cylinder)

$= \pi \times (3)^2 \times h$

$= 9\pi h \ m^2$

Area of embankment $= \pi R^2 - \pi r^2$

Where R = r + width of the embankment

$= 3 + 2$

$R = 5$ m

Area of embankment $= \pi 5^2 - \pi 3^2$

$= 16\pi \ m^2$

Volume of earth dug out for the embankment

= area of embankment × height

$9\pi h = 16\pi \times 2.25$

$h = \frac{16\pi \times 2.25}{9\pi}$

$\therefore h = 4$ m

$\therefore$ The depth of the well is 4 m.

Solution 25.

Inner diameter of base = 20 cm

Radius = 10 cm

Area of the wet surface of the cylinder

= Inner curved surface area of cylinder + area of base

$= 2\pi rh + \pi r^2$

$= 2\pi \times 10 \times 14 + \pi(10)^2$

$= 280\pi + 100\pi$

$= 380\pi$

$= 1194 \ cm^2$

$\therefore$ Area of the wet surface of the cylinder is 1194 cm^2.

Solution 26.

Initial values :

radius = r

height = h

$$\text{Volume}(V_1) = \pi r^2 h$$

$$\text{Curved surface area}(C_1) = 2\pi rh$$

New values after change:

$$\text{radius} = r - 10\% \text{ of } r$$
$$= r - 0.1r$$
$$= 0.9r$$

$$\text{height} = h + 20\% \text{ of } h$$
$$= h + 0.2h$$
$$= 1.2h$$

$$\text{Volume } (V_2) = \pi(0.9r)^2 \times 1.2h$$
$$= 0.972\pi r^2 h$$

$$\text{Curved surface area}(C_2) = 2\pi(0.9r)(1.2h)$$
$$= 2\pi rh\ (1.08)$$

$$\text{Change in percentage of Volume} = \frac{(V_1 - V_2)}{V_1} \times 100$$
$$= \frac{(\pi r^2 h - 0.972\pi r^2 h)}{\pi r^2 h} \times 100$$
$$= \frac{\pi r^2 h(1 - 0.972)}{\pi r^2 h} \times 100$$
$$= 0.028 \times 100$$
$$= 2.8\%$$

The positive value indicates that V_1 is greater than $V_2 (V_1 - V_2)$, which indicates that there is a decrease in volume by 2.8%. (Ans 1)

Change in the percentage of the

$$\text{Curved surface area} = \frac{(C_1 - C_2)}{C_1} \times 100$$
$$= \frac{(2\pi rh - 2\pi rh \times 1.08)}{2\pi rh} \times 100$$
$$= \frac{2\pi rh(1 - 1.08)}{2\pi rh} \times 100$$
$$= -0.08 \times 100 = 8\%$$

(negative sign indicates that C_1 is smaller than C_2)

$\Rightarrow$ There is an 8% increase in the curved surface area.

Solution 27.

Inner radius of tap = 0.8 cm

Circular area = πR^2

$= \pi(0.8)^2$

$= 0.64\pi \text{ cm}^2$

Rate of water flow = 7 m/s = 700 cm/s

Volume of water flowing out of the tap in one second

= rate of flowing of water x circular area of tap

$= 700 \times 0.64\pi$

$= 700 \times 0.64 \times 3.142$

$= 1408 \text{ cm}^3$

So volume of water flowing out in 75 minutes i.e 75 x 60 s

$= 1408 \times 75 \times 60 \text{ cm}^3$

$= 6336000 \text{ cm}^3$

$= \frac{6336000}{1000}$ litres

= 6336 litres

∴ Volume of water delivered by the pipe is 6336 litres.

Solution 28.

For the cylindrical tank:

diameter = 4m
radius = 2m
= 200 cm

Height = 6m
= 600 cm

Volume of the cylindrical tank = $\pi r^2 h$

$= \pi(200)^2 \times 600$

$= 24000000\pi \text{ cm}^3$

For the cylindrical pipe:

diameter = 4 cm
radius = 2 cm
rate of flow of water = 10 m/s = 1000 cm/s

Volume of water flown in 1 sec = area of base x rate of flow of water

$= \pi \times 2^2 \times 1000$

$= 4000\pi \text{ cm}^3$

$\therefore$ Time taken to fill the tank $= \dfrac{24000000\pi}{4000\pi}$

$= \dfrac{24000}{4}$

$= 6000$ s

$= \dfrac{6000}{60}$ mins

$= 100$ minutes

$\therefore$ The time taken to fill the tank is 100 mins.

Solution 29.

Curved surface area of cylinder = $\pi r^2 h$

Height h = 14 cm

The difference between the outer and inner curved surface area = 264 cm ^{2}

$2\pi \times \{(R_{outer})-(R_{in})\} \times h = 264 \text{ cm}^2$

$\Rightarrow \quad \pi \times \{(R_{outer})-(R_{in})\} \times h = 132 \qquad (1)$

Volume of material in cylinder = 1980 cm^3

$\pi\{(R_{outer})^2-(R_{in})^2\} \times h = 1980$

$\pi\{(R_{outer})-(R_{in})\}\{(R_{outer}+(R_{in})\} \times h = 1980 \quad (2)$

Substituting (1) in (2), we get

$\{(R_{outer}+(R_{in})\} \times 132 = 1980$

$\therefore \{(R_{outer}+R_{in}\} = 15 \text{ cm}$

Total surface area of a hollow cylinder

$= 2\pi\{(R_{outer}+R_{in}\} \times h + 2\pi\{(R_{outer})^2-(R_{in})^2\}$

$= 2\pi \times 15 \times 14 + 2 \times \dfrac{1980}{14}$

$= 1602 \text{ cm}^2$

$\therefore$ The total surface area of the hollow cylinder is 1602 cm^2.

Solution 30.

Let height = h

radius = r

$h + r = 28$ cm (given)

$\therefore h = 28 - r$ cm

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

$$2\pi r(28-r) + 2\pi r^2 = 616$$

$$56\pi r - 2\pi r^2 + 2\pi r^2 = 616$$

$$56\pi r = 616$$

$$r = \frac{616}{56\pi}$$

$$r = 3.5 \text{ cm}$$

$$h = 28 - r$$

$$= 28 - 3.5$$

$$= 24.5 \text{ cm}$$

Volume of cylinder = $\pi r^2 h$

$$= \pi(3.5)^2 \times 24.5$$

$$= 943 \text{ cm}^3$$

$\therefore$ Volume of cylinder is 943 cm^3.

Solution 31.

Internal diameter of tube = 10.4 cm

Internal radius = 5.2 cm

Length of tube = 25 cm

Thickness of metal = 8 mm

= 0.8 cm

Outer radius = Internal radius + Thickness

= 5.2 + 0.8

= 6 cm

Volume of metal = Volume of material between outer radius and inner radius

$= \pi(R^2 - r^2)h$

$= \pi\{6^2 - (5.2)^2\} \times 25$

$= 704 \text{ cm}^3$

∴ The volume of the metal used is 704 cm^3. (Ans 1)

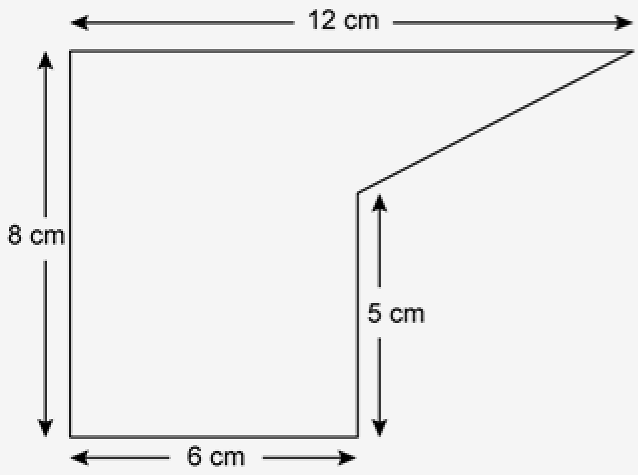
1 cm^3 of metal = 1.42g

∴ 704 cm^3 of metal = 704×1.42

= 999.68 g

∴ The weight of the tube is 999.68 g. (Ans 2)

Ex No: 25.3
Solution 1.



(a) Divide the figure into 1 rectangle and 1 triangle.

Dimensions of the rectangle:
length = 8 cm
breadth = 6 cm

Area of rectangle = length x breadth
= 8 x 6
= 48 cm^2 (1)

Dimensions of the triangle:

base = 12-6
= 6 cm
height = 8-5
= 3 cm

Area of a triangle = $\frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times 3$

$= 9\ cm^2$ (2)

Area of the cross section = 48+9

$= 57\ cm^2$

$\therefore$ Area of the cross section is $57\ cm^2$.

(b)

Length (height) of the metal = 2m

= 200cm

Volume of the metal = Area of cross-section x height

$= 57 \times 200$

$= 11400\ cm^3$

$\therefore$ Volume of the metal is $11400\ cm^3$.

Solution 2.

The given figure is a trapezium because 2 opposite sides are parallel.

length of the pool = 40 m

height of the trapezium = 10 m

Area of cross section = Area of trapezium

$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$= \frac{1}{2} \times (2+3) \times 10$

$= 25\ m^2$

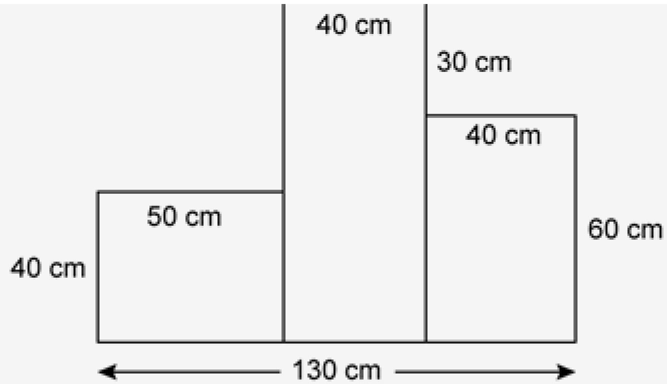
Volume of the pool = Area of cross section x length

$= 25 \times 40$

$= 1000\ m^3$

$\therefore$ The volume of the pool is $1000\ m^3$.

Solution 3.



(a) To find the volume, first find the area of the figure.
To find the area, we divide the figure into 3 different rectangles.

Rectangle 1 (left):

length = 50 cm
width = 40 cm
Area = length x width
= 50 x 40
= 2000 cm^2

Rectangle 2 (middle):

length = (60 + 30) cm = 90 cm
width = 40 cm
Area = length x width
= 90 x 40
= 3600 cm^2

Rectangle 3 (right):

length = 60 cm
width = 40 cm
Area = length x width
= 60 x 40
= 2400 cm^2

Total area = 2000+3600+2400

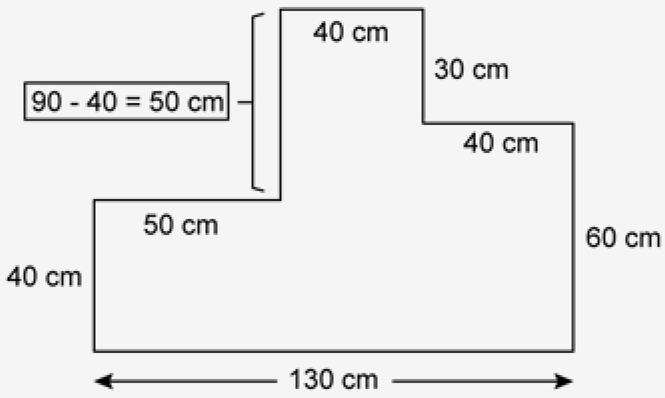
$= 8000 \text{ cm}^2$

Volume = Total area x length

$= 8000 \times 60$

$= 4{,}80{,}000 \text{ cm}^3$

$\therefore$ The space occupied is $4{,}80{,}000 \text{ cm}^3$.



(b) Total surface area = 2 x Area of cross section+
Area of bottom face+Area of left face+
Area of right face+Area of top face

Area of cross-section = 8000 cm^2 (from a)

Width of the stand = 60 cm (given)

Area of bottom face = 130×60

$= 7800 \text{ cm}^2$

Area of the left face = $40 \times 60 + 50 \times 60 + 50 \times 60$

$= 8400 \text{ cm}^2$

Area of right face = $60 \times 60 + 40 \times 60 + 30 \times 60$

$= 7800 \text{ cm}^2$

Area of the top face = 40×60

$= 2400 \text{ cm}^2$

Total area = 2000+3600+2400

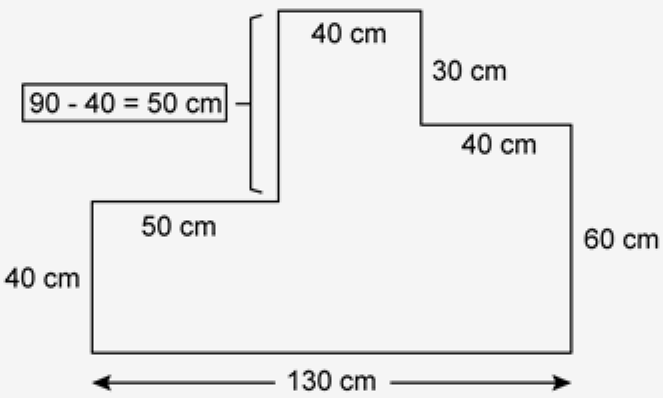
$= 8000\ cm^2$

Volume = Total area x length

= 8000 x 60

$= 4{,}80{,}000\ cm^3$

∴ The space occupied is $4{,}80{,}000\ cm^3$.



(b) Total surface area = 2 x Area of cross section+
Area of bottom face+Area of left face+
Area of right face+Area of top face

Area of cross-section = $8000\ cm^2$ (from a)
Width of the stand = 60 cm (given)

Area of bottom face = 130 x 60

$= 7800\ cm^2$

Area of the left face = 40 x 60+50 x 60+50 x 60

$= 8400\ cm^2$

Area of right face = 60 x 60+40 x 60+30 x 60

$= 7800\ cm^2$

Area of the top face = 40 x 60

$= 2400\ cm^2$

$$\text{Total surface area} = 2 \times 8000 + 7800 + 8400 + 7800 + 2400$$
$$= 42400 \text{ cm}^2$$
$$= 4.24 \text{ m}^2$$

$\therefore$ The total surface area is 4.24 m^2.

Solution 4.

$$\text{Rate of flow of water} = 1.5 \text{ m/s}$$
$$= 150 \text{ cm/s}$$

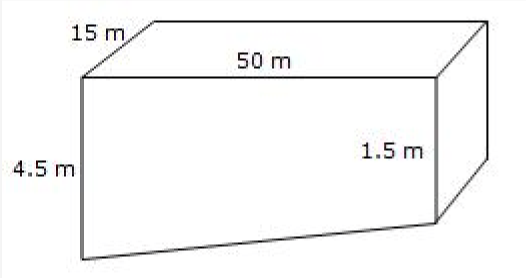
$$\text{Rate of volume of water flown} = \text{Rate of flow} \times \text{cross section area}$$
$$= 150 \times 2.5$$
$$= 375 \text{ cm}^3/\text{s}$$

$$\text{Total volume of water flow} = \text{Rate of volume of water flown} \times \text{Time}$$
$$= 375 \times (4 \times 60 \text{ seconds})$$
$$= 90000 \text{ cm}^3$$

$$\text{Volume of water flown} = \text{Volume of water in the tank}$$
$$90000 = l \times b \times h$$
$$90000 = 90 \times 50 \times h$$
$$h = 20 \text{ cm}$$

$\therefore$ The rise in the level of water is 20 cm.

Solution 5.



Area of cross section = Area of trapezium

$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$= \frac{1}{2} \times (1.5 + 4.5) \times 50$

$= \frac{1}{2} \times 6 \times 50$

$= 150 \text{ m}^2$

$\therefore$ Volume of the pool = area of cross section x height

$= 150 \times 15$

$= 2250 \text{ m}^3$

$(\because 1 \text{ m}^3 = 1 \text{ kilolitre})$

$\therefore$ Volume of the pool is 2250 kilolitres.

Solution 6.

Dimensions of the tank:

length = 90 m
breadth = 70 cm
= 0.7 m
height = 84 cm
= 0.84 m

$$\text{Amount of rainfall} = \frac{\text{Volume of tank}}{\text{Area of roof}}$$
$$= \frac{90 \times 0.7 \times 0.84}{28 \times 9}$$
$$= 0.21 \text{ m}$$
$$= 21 \text{ cm}$$

$\therefore$ Amount of rainfall is 21 cm.

Solution 7.

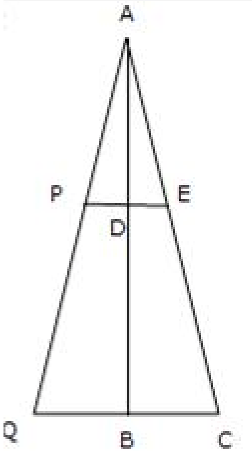
$$1 \text{ km/hr} = \frac{1 \text{ km}}{1 \text{ hr}}$$
$$= \frac{100000 \text{ cm}}{3600 \text{ s}}$$
$$= \frac{250}{9} \text{ cm / s}$$

Rate of flow of water = Area of cross section x rate
$= 5.4 \times 27$ km/hr
$= 5.4 \times 27 \times 250/9$ cm/s
$= 4050 \text{ cm}^3\text{/s}$

Volume of water flown out in 2 minutes
= Rate of flow of water x Time
$= 4050 \times (2 \times 60 \text{ seconds})$
$= 486000 \text{ cm}^3$
$= \frac{486000}{1000}$
$= 486$ litres ($\because$ 1 litre $= 1000 \text{ cm}^3$)

$\therefore$ Volume of water which flows out of the pipe in 2 minutes is 486 liters.

Solution 8.



(a) Complete the diagram as shown:

Let AD = x m

$AB = AD + DB$

$= (x+4)$ m

$BC = \frac{1}{2} \times QC$

$= \frac{1}{2} \times 2$

$= 1$ m

$DE = \frac{1}{2} \times PE$

$= \frac{1}{2} \times 0.2$

$= 0.1$ m

In ΔADE and ΔABC,

$\angle ADE = \angle ABC$ (90^0each)

$\angle DAE = \angle BAC$ (Common angle)

$\therefore \Delta ADE \sim \Delta ABC$ by AA test

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ (C.S.S.T.)

$\frac{x}{x+4} = \frac{0.1}{1}$

$\frac{10x}{x+4} = \frac{10 \times 0.1}{1}$ Multiply by 10 on both sides

$10x = x+4$

$9x = 4$

$x = \frac{4}{9}$ m

$\therefore AB = \frac{4}{9} + 4$

$= \frac{40}{9}$ m

Area of the cross section of the wall

$= A(\Delta AQC) - A(\Delta APE)$

$= \frac{1}{2} \times QC \times AB - \frac{1}{2} \times PE \times AD$

$= \frac{1}{2} \times 2 \times \frac{40}{9} - \frac{1}{2} \times 0.2 \times \frac{4}{9}$

$= \frac{40}{9} - \frac{0.4}{9}$

$= \frac{39.6}{9}$

$= 4.4$

$\therefore$ The area of the cross section of the wall is 4.4 sq. m

(b) Area of the cross section of the wall = 4.4 sq. m .... from (a)

Volume of the wall = Area of the cross section $\times$ length

$= 4.4 \times 40$

$= 176$ m^3

$\therefore$ Volume of the wall is 176 m^3.

(c) Cost for painting will depend on the total surface area which includes 5 faces (2 cross sectional, 2 lateral rectangles and 1 top face)

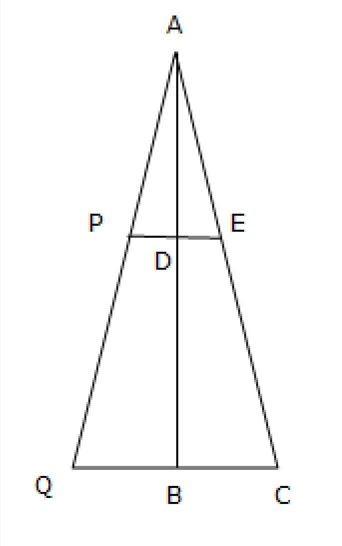
Area of 1 cross section = 4.4 m^2....from (a)
Area of 2 cross sections = 2×4.4

$= 8.8\ m^2$

To find the area of the rectangles, we need to first find length of side PQ.

$$PQ = AQ - AP$$

By applying Pythagoras theorems in ΔABQ and ΔAPD



In ΔABQ,

$AQ^2 = AB^2 + QB^2$

$= \left(\frac{40}{9}\right)^2 + 1^2$

$= \frac{1600}{81} + 1$

$= \sqrt{\frac{1681}{81}}$

$= \frac{41}{9}$

AQ = 4.56 m

In ΔAPD,

$AP^2 = AD^2 + PD^2$

$= (\frac{4}{9})^2 + 0.1^2$

$= \frac{16}{81} + 0.01$

$= \sqrt{\frac{16.81}{81}}$

$= \frac{4.1}{9}$

$AP = 0.46$ m

$PQ = AQ - AP$

$= 4.56 - 0.46$

$= 4.1$ m

Total surface area of 5 faces

= 2 x Area of cross section+

Area of 2 lateral faces+Area of top face

$= 2 \times 4.4 + 2 \times PQ \times \text{length} + 0.2 \times 40$

$= 8.8 + 328 + 8$

$= 344.8 \text{ m}^2$

Cost of painting $= 344.8 \times 2.50$

= Rs. 862

∴ The cost of painting the wall is Rs. 862.

Solution 9.

(a) The internal surface area will consist of faces formed by 1 side as length and other sides as AD, CD and BC.

$AM = \frac{1}{2}(AB - CD)$

$= \frac{1}{2}(4.4 - 3)$

$AM = 0.7$

In ΔAMD, by Pythagoras theorem,

$AD^2 = AM^2 + DM^2$

$AD^2 = (0.7)^2 + (2.4)^2$

$AD = 2.5m$

$AD = BC = 2.5\ m$

Total surface area = (length x AD)+(length x CD)+(length x BC)

= 5.4(AD+CD+BC)

= 5.4(2.5+3+2.5)

= 43.2 m^2

Cost of painting = 43.2 x 5

= Rs 216

$\therefore$ The cost of painting the internal surface is Rs. 216.

(b) Flooring will be done on an area formed by AB and length.

Area of floor = AB x length

= 4.4 x 5.4

= 23.76 m^2

$\therefore$ The cost of flooring = 2.5 x 23.76

= Rs 59.4

Solution 10.

(a) Volume of shed = Area of wall $\times$ length

Area of the wall

$= \text{Area of } \Delta CDE + \text{Area of rectangle } ABCE$

$= \frac{1}{2} \times \text{base} \times \text{height} + AB \times AE$

$= \frac{1}{2} \times 8 \times 3 + 8 \times 7.5$

$= 72 \text{ m}^2$

$\therefore$ The volume of the shed = 72×50

$= 3600 \text{ m}^3$ (Ans 1)

(b) Asbestos sheets are spread on the area formed by the rectangle with CD and DE as lengths.

In ΔCDE, by Pythagoras theorem,

$DE^2 = (\text{perpendicular})^2 + (\frac{AB}{2})^2$

$DE^2 = 3^2 + (\frac{8}{2})^2$

$DE^2 = 3^2 + 4^2$

$DE^2 = 25$

$\therefore DE = CD = 5 \text{ m}$

Area of asbestos sheets = DE $\times$ length + DC $\times$ length

Area of asbestos sheet = $2 \times 5 \times 50$

$= 500 \text{ m}^2$

Cost of roofing = Area $\times$ rate

$= 500 \times 20$

= Rs 10,000

(c)

Total area = 2 x Area of asbestos + 2 x Area of wall + 2 x AE x length

= 500 + 2 x 72 + 2 x 7.5 x 50 (from a and b)

= 500 + 144 + 750

= 1394 m^2

∴ The total surface area of the shed is 1394 m^2.

Solution 11.

Area of cross-section = Area of trapezium

$= \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$= \frac{1}{2} \times (1 + 3) \times 1.5$

$= 3\ m^2$

Volume of the pool = Area of cross-section x length

$= 3 \times 50$

$= 150\ m^3$

∴ The volume of the pool is 150 m^3.

Solution 12.

Water delivered in 10 mins = 1800 liters

$= 1800 \times 1000 \text{ cm}^3$

Volume of water = Speed of water (in cm/min) × Area of cross-section × Time

$1800000 = \text{Speed} \times 3 \times 10$

$\therefore \text{Speed} = 60000 \text{ cm/min}$

$$1 \frac{\text{cm}}{\text{min}} = \frac{1 \div 100000 \text{ km}}{1 \div 60 \text{ hr}} = \frac{60}{100000} = \frac{6}{10000} \text{km/hr}$$

$$\therefore \text{Speed} = 60000 \times \frac{6}{10000} \text{km/hr} = 36 \text{ km/hr}$$

∴ The speed of the water through the pipe is 36 km/hr.

Solution 13.

$$\text{Area of trapezoid} = \frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (3+5) \times 2$$

$$= 8 \text{ m}^2$$

$$\text{Volume of the canal} = \text{Area of trapezoid} \times \text{Length} = 8 \times 400 = 3200 \text{ m}^3$$

∴ The volume of water that it holds is 3200 m^3.

Solution 14.

Time taken to fill the tank

$$= \frac{\text{Volume of the tank}}{\text{Volume of water flown in it in 1 s}}$$

Volume of tank = $800 \times 600 \times 4 \text{ cm}^3$

Volume of water flown out of the pipe in 1 s

= Area of cross section × Rate of flow

= 1.5×10 (m/s)

= $1.5 \times 10 \times 100$ (cm/s)

= 1500 cm

$\therefore$ The time taken to fill the tank = $\frac{800 \times 600 \times 4}{1500}$

= 1280 s

$\therefore$ The time taken to fill the tank is 1280 s.

Solution 15.

Volume of water flown = Area × Time × Rate

= $6 \times 3600\text{s} \times 30 \text{ cm}^3/\text{s}$

= 648000 cm^3

= 648 liters

($\because$ 1 liter = 1000 cm^3)

$\therefore$ The volume of water flown is 648 liters.